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Cosmic Strings and Domain Walls in a Scale-Covariant Theory of Gravitation

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Abstract Axially symmetric space-time is considered in the presence of cosmic string source and thick domain walls in the frame work of a scale-covariant theory of gravitation. A relation between metric potential is assumed to get a determinate solution of the field equations of this theory. In this particular case, it is observed that the geometric (Nambu) string p-string (Takabayasi string) and Reddy string do not survive. It is also seen that the stiff (self-gravitating) domain walls do not exist in this theory.

Keywords Cosmic strings · Domain walls · Scale-covariant theory

1 Introduction

Alternative theories of gravity have been extensively studied in connection with their cosmological applications. Noteworthy among them are scalar-tensor theories of gravitation formulated by Brans and Dicke [1], Nordvedt [2] and Saez and Ballester [3]. In the Brans–Dicke theory there exists a variable gravitational parameter. Canuto et al. [4] formulated a scale-covariant theory of gravitation which also admits a variable G and which is a viable alternative to general relativity [5, 6]. In the scale-covariant theory Einstein's field equations are valid in gravitational units where as physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by a conformal transformation

$$\overline{g}_{ij} = \phi^2(x^k) g_{ij} \tag{1}$$

where in Latin indices take values 1, 2, 3, 4 bars denote gravitational units and unbar denotes atomic quantities. The gauge function ϕ $(0 < \phi < \infty)$ in its most general formulation is a function of all space-time coordinates. Thus, using the conformal transformation of the type

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given by (1), Canuto et al. [4] transformed the usual Einstein equations into

$$R_{ij} - 1/2Rg_{ij} + f_{ij}(\phi) = -8\pi G(\phi)T_{ij} + \Lambda(\phi)g_{ij} \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}(\phi\phi^k_{;k} - \phi^k\phi_k). \tag{3}$$

Here R_{ij} is the Ricci tensor, R the Ricci scalar, Λ the cosmological 'Constant', G the gravitational 'constant' and T_{ij}, the energy momentum tensor. A semicolon denotes covariant derivative and ϕ_i denotes ordinary derivative with respect to x^i. A particular feature of this theory is that no independent equation for ϕ exists. The possibilities that have been considered for gauge function ϕ are [4, 7]

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon}, \quad \varepsilon = \pm 1, \pm 1/2 \tag{4}$$

where t_0 is constant. The form

$$\phi \sim t^{1/2}$$

is the one most favored to fit observations [8, 9].

Also, the energy conservation equation for perfect fluid.

$$\rho_4 + (\rho + p)u^i_{;i} = -\rho_4 + \frac{(G\phi)_4}{G\phi} - 3p\frac{\phi_4}{\phi} \tag{5}$$

is a consequence of the field equations (3) (Eq. (4) of Canuto et al. [4, 7]).

Canuto et al. [7], Beesham [10–12], Reddy and Venkateswarlu [13], Reddy et al. [14] and Reddy and Venkateswarlu [15] have investigated several aspects of this theory of gravitation with the perfect fluid matter distribution as source.

In recent years there has been a lot of interest in the study of alternative theories of gravitation in the presence of cosmic string source and thick domain walls. Cosmic strings and domain walls are the topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe. A systematic survey of cosmic strings and other topological defects is presented by Vilenkin and Shellard [16].

The gravitational effects of comic strings, both in general relativity and in the alternative theories of gravitation, have been extensively discussed by Vilenkin [17], Gott [18], Letelier [19], Stachel [20], Krori et al. [21], Banerjee et al. [22], Tikekar and Patel [23], Tikekar et al. [24], Rahaman et al. [25], Reddy [26, 27] and Reddy [28, 29]. So far a considerable amount of work has also been done on domain walls. Vilenkin [30], Isper and Sikivie [31], Widrow [32], Gotez [33], Mukherji [34] and Wang [35] are some of the authors who have investigated several aspects of domain walls in general relativity. Recently, Rahaman et al. [36], Rahaman [37], Rahaman and Mukherji [38] and Reddy and Rao [39] have discussed thick domain walls in Lyra [40] geometry. It is evident from the literature that axially symmetric space-time in the presence of cosmic string source and thick domain walls has not been considered, so far, in scale covariant theory of gravitation. Hence, in this paper, we investigate the axially symmetric cosmic strings and thick domain walls in the scale-covariant theory of gravitation.

2 Cosmic Strings

In this section, we discuss the non-existence of axially symmetric cosmic strings in the scale-covariant theory of gravitation. Here we consider the energy-momentum tensor for cosmic string source as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{6}$$

where ρ is the rest energy density of the cloud of strings with massive particles attached to them, $\rho = \rho_p + \lambda$, ρ_p being the rest energy of the particles attached to the strings and λ the tension density of the system of strings. As pointed out by Letelier [19], λ may be positive or negative, u^i describes the cloud four-velocity and x^i represents the direction of anisotropy, i.e., the direction of strings.

We consider the axially symmetric space-time metric given by [41]

$$ds^2 = dt^2 - A^2(t)[d\chi^2 + F^2(\chi)d\Phi^2] - B^2(t)dz^2 \tag{7}$$

with the convention $X^1 = \chi$, $X^2 = \Phi$, $X^3 = z$, $X^4 = t$ and A and B are functions of the proper time t alone while F is a function of the coordinate χ alone. Orthonormalisation of u^i and x^i is given as

$$u^i u_i = 1, \qquad x^i x_j = 0, \qquad x^i x_i = -1. \tag{8}$$

In the commoving coordinate system, we have from (6)

$$T_4^4 = \rho, \qquad T_1^1 = T_2^2 = 0, \qquad T_3^3 = \lambda, \qquad T_j^i = 0 \quad \text{for } i \neq j. \tag{9}$$

The quantities ρ and λ depend on t only. Here the string source is along the z-axis which is the axis of symmetry.

Now, with the help of (6), (8) and (9) the field equations (2), (3) and (5) for the metric (7), with zero cosmological 'constant', can be written as

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{\phi_{44}}{\phi} + \frac{B_4\phi_4}{B\phi} - \frac{\phi_4^2}{\phi^2} = 0, \tag{10}$$

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 - \frac{1}{A^2}\left(\frac{F_{11}}{F}\right) + \frac{\phi_{44}}{\phi} + 2\frac{A_4\phi_4}{A\phi} - \frac{B_4\phi_4}{B\phi} - \frac{\phi_4^2}{\phi^2} = 8\pi G\lambda, \tag{11}$$

$$\frac{1}{A^2}\left(\frac{F_{11}}{F}\right) - \left(\frac{A_4}{A}\right)^2 - 2\frac{A_4 B_4}{AB} + \frac{\phi_{44}}{\phi} - 2\frac{A_4\phi_4}{A\phi} - \frac{B_4\phi_4}{B\phi} - 3\frac{\phi_4^2}{\phi^2} = -8\pi G\rho, \tag{12}$$

$$\rho_4 + \rho\left(\frac{2A_4}{A} + \frac{B_4}{B}\right) + \lambda\frac{B_4}{B} = -\rho\left(\frac{G_4}{G}\right) - (\rho - \lambda)\frac{\phi_4}{\phi} \tag{13}$$

where the suffix 1 and 4 after an unknown function denote differentiation with respect to χ and t respectively.

The functional dependence of the metric together with (11) and (12) imply $\frac{F_{11}}{F} = k^2$, $k^2 = \text{constant}$.

If $k = 0$, then $F(\chi) = (\text{constant})\ \chi$, $0 < \chi < \infty$. This constant can be made equal to 1 by suitably choosing units for ϕ. Thus we shall have

$$f(\chi) = \chi \tag{13a}$$

resulting in the flat model of the universe [42]. Now the field equations (11) and (12) reduce to

$$2\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2+\frac{\phi_{44}}{\phi}+2\frac{A_4\phi_4}{A\phi}-\frac{B_4\phi_4}{B\phi}-\frac{\phi_4^2}{\phi^2}=8\pi G\lambda, \tag{14}$$

$$\left(\frac{A_4}{A}\right)^2+2\frac{A_4B_4}{AB}-\frac{\phi_{44}}{\phi}+2\frac{A_4\phi_4}{A\phi}+\frac{B_4\phi_4}{B\phi}+3\frac{\phi_4^2}{\phi^2}=-8\pi G\rho. \tag{15}$$

The field equations (10), (13), (14) and (15) are four equations in six unknowns A, B, ϕ, ρ, λ and $G(\phi)$. Hence to get a determinate solution we assume a relation between metric potentials [41]

$$A=\alpha B,\quad \alpha=\text{constant} \tag{16}$$

and the equations of state of string model [19]

$$\rho=\lambda\quad \text{(geometric or Nambu string)}, \tag{17}$$

$$\rho=(1+\omega)\lambda\quad \text{(p-string or Takabayasi string)}, \tag{18}$$

and

$$\rho+\lambda=0\quad \text{(Reddy string [26, 27, 39])}. \tag{19}$$

Equations (10) and (14) on subtraction yield

$$\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2-\frac{B_{44}}{B}-\frac{A_4B_4}{AB}+2\frac{A_4\phi_4}{A\phi}-2\frac{B_4\phi_4}{B\phi}=8\pi G\lambda. \tag{20}$$

Using (16) in (19) we get

$$\lambda=0. \tag{21}$$

Using (20) in (17), (18) and (19) we get $\rho=0$, which shows that in scale covariant theory neither geometric (Nambu) string nor the p-string (Takabayasi string) survive. Hence we observe that the geometric strings, p-strings and Reddy string do not exist in the scale covariant theory of gravitation.

3 Thick Domain Walls

In this section we discuss the thick domain walls in the axially symmetric space time given by (7). A thick domain wall can be viewed as a soliton like solution of the scalar field equations coupled with gravity. There are two ways of studying thick domain walls. One way is to solve gravitational field equations with an energy-momentum tensor describing a scalar field Ψ with self interactions contained in a potential $V(\Psi)$ given by

$$\Psi_i\Psi_j-g_{ij}[1/2\Psi_k\Psi^k-V(\Psi)].$$

Second approach is to assume the energy momentum tensor in the form

$$T_{ij}=\rho(g_{ij}+w_iw_j)+pw_iw_j,\quad w^iw_j=-1 \tag{22}$$

where ρ is the energy density of the walls, p is the pressure in the direction normal to the plane of the wall and w_i is a unit space-like vector in the same direction [36].

Here we use the second approach to study the thick domain walls in scale covariant theory of gravitation. We, again, consider the axially symmetric metric given by (7). In the commoving coordinate system we have from (21).

$$T_4^4 = T_1^1 = T_2^2 = \rho, \qquad T_3^3 = -p, \qquad T_j^i = 0 \quad \text{for } i \neq j. \tag{23}$$

Here pressure is taken in the direction of z-axis. The quantities ρ and p depend on t only.

Now, the field equations (2), (3) and (5) (with zero cosmological 'constant') for the metric (7), with the help of (21), (22) and (13a) can be written as

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{\phi_{44}}{\phi} + \frac{B_4\phi_4}{B\phi} - \frac{\phi_4^2}{\phi^2} = 8\pi G\rho, \tag{24}$$

$$2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 + \frac{\phi_{44}}{\phi} + 2\frac{A_4\phi_4}{A\phi} - \frac{B_4\phi_4}{B\phi} - \frac{\phi_4^2}{\phi^2} = -8\pi Gp, \tag{25}$$

$$\left(\frac{A_4}{A}\right)^2 + 2\frac{A_4 B_4}{AB} - \frac{\phi_{44}}{\phi} + 2\frac{A_4\phi_4}{A\phi} + \frac{B_4\phi_4}{B\phi} + 3\frac{\phi_4^2}{\phi^2} = 8\pi G\rho, \tag{26}$$

$$\rho_4 + (\rho + p)\frac{B_4}{B} = -\rho\left(\frac{G_4}{G}\right) - (\rho + p)\frac{\phi_4}{\phi}. \tag{27}$$

Equations (23–26) are a set of three independent equations in five unknowns A, B, p, ρ and $G(\phi)$. Hence to get a determinate solution assume a relation between metric potentials given by (15). We also assume the equation of state

$$\rho = p \tag{28}$$

which represents self-gravitating or stiff domain walls. Now, subtracting (23) from (24) we obtain

$$8\pi G(\rho + p) = \frac{B_{44}}{B} - \frac{A_{44}}{A} + \frac{A_4 B_4}{AB} - \left(\frac{A_4}{A}\right)^2 + 2\frac{B_4\phi_4}{B\phi} - 2\frac{A_4\phi_4}{A\phi} \tag{29}$$

using (16) in (28), we have

$$\rho + p = 0 \quad \text{or} \quad p = -\rho \tag{30}$$

which leads to domain wall models in the accelerated universe dominated by a fluid with negative pressure such that the strong energy condition is violated [42]. However since in the standard cosmology a fluid of negative pressure violating the strong energy condition does not cluster at large scale in the relativistic regime equation (29) and (27) together yield

$$\rho = 0 = p \tag{31}$$

which shows that, stiff or self-gravitating domain walls do not survive in scale covariant theory of gravitation in this particular case. It may be mentioned here that Reddy and Rao [39] have presented axially symmetric cosmological models, which represent geometric string and stiff domain walls in Lyra [40] geometry.

4 Conclusions

The origin of structure in the universe is one of the greatest cosmological mysteries even today. The exact physical situation at very early stages of the formation of our universe is still known. Axially symmetric string models and domain walls play a vital role in understanding the formation of large scale structures in the universe Bhattacharya and Karade [41] presented axially symmetric Nambu string and Takabayasi string (p-string) models in general relativity. In this paper, we have shown that axially symmetric cosmic string models which represent Nambu string (geometric string) p-string [19] and Reddy string [39] do not survive in scale covariant theory of gravitation formulated by Canuto et al. [4, 7] when we assume a relating between metric coefficients. We have also shown, in this particular case, that self-gravitating or stiff domain walls do not exist.

References

1. Brans, C.H., Dicke, R.H.: Phys. Rev. **124**, 925 (1961)
2. Nordvedt, K., Jr.: Astrophys. J. **161**, 1069 (1970)
3. Saez, D., Ballester, V.J.: Phys. Lett. A **1113**, 467 (1985)
4. Canuto, V.M., Hsieh, S.H., Adams, P.J.: Phys. Rev. Lett. **39**, 429 (1977)
5. Wesson, P.S.: Gravity, Particles and Astrophysics. Reidel, Dordrecht (1980)
6. Will, C.M.: Phys. Rep. **113**, 345 (1984)
7. Canuto, V.M., Hsieh, S.H., Tsiang, E.: Phys. Rev. D **16**, 1643 (1977)
8. Canuto, V.M., Goldman, I.: Nature **304**, 311 (1983)
9. Canuto, V.M., Goldman, I.: In: Abell, G.O., Chincarini, G. (eds.) Early Evolution of the Universe and Its Present Structure, p. 485. Reidel, Dordrecht (1983)
10. Beesham, A.: Class. Quantum Gravity **3**, 481 (1986)
11. Beesham, A.: Astrophys. Space Sci. **119**, 415 (1986)
12. Beesham, A.: Class. Quantum Gravity **3**, 1027 (1986)
13. Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **136**, 191 (1987)
14. Reddy, D.R.K., Murthy, Ch.S.V.V.R., Venkateswarlu, R.: J. Ind. Math. Soc. **68**, 179 (2002)
15. Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **289**, 1 (2004)
16. Vilenkin, A., Shellard, E.P.S.: Cosmic Strings and other Topological Defects. Cambridge University Press, Cambridge (1994)
17. Vilenkin, A.: Phys. Rev. D **23**, 852 (1981)
18. Gott, J.R.: Astrophys. J. **288**, 422 (1985)
19. Letelier, P.S.: Phys. Rev. D **28**, 2414 (1983)
20. Satchel, J.: Phys. Rev. D **21**, 2171 (1980)
21. Krori, K.D., Chaudhuri, T., Mahanta, C.R., Majumdar, A.: Gen. Relativ. Gravit. **22**, 123 (1990)
22. Banerjee, A., Samyal, A.K., Chakraborti, S.: Pramana J. Phys. **34**, 1 (1990)
23. Tikekar, R., Patel, L.K.: Gen. Relativ. Gravit. **26**, 647 (1990)
24. Tikekar, R., Patel, L.K., Dadhich, N.: Gen. Relativ. Gravit. **26**, 647 (1994)
25. Rahaman, F., Chakraborti, S., Hossain, M., Begum, N., Bera, J.: Ind. J. Phys. B **76**, 747 (2002)
26. Reddy, D.R.K.: Astrophys. Space Sci. **286**, 359 (2003)
27. Reddy, D.R.K.: Astrophys. Space Sci. **286**, 365 (2003)
28. Reddy, D.R.K.: Astrophys. Space Sci. **300**, 381 (2005)
29. Reddy, D.R.K.: J. Dyn. Syst. Geom. Theor. **3**, 197 (2005)
30. Vilenkin, A.: Phys. Lett. B **133**, 177 (1983)
31. Isper, J., Sikivie, P.: Phys. Rev. D **30**, 712 (1984)
32. Widrow, L.M.: Phys. Rev. D **39**, 3571 (1989)
33. Goetz, G.: J. Math. Phys. **31**, 2683 (1990)
34. Mukherji, M.: Class. Quantum Gravity **10**, 131 (1993)
35. Wang, A.: Mod. Phys. Lett. **39**, 3605 (1994)
36. Rahaman, F., Chakraborty, S., Kalam, M.: Int. J. Mod. Phys. D **10**, 735 (2001)
37. Rahaman, F.: Astrophys. Space Sci. **282**, 625 (2002)

38. Rahaman, F., Mukherji, R.: Astrophys. Space Sci. **288**, 493 (2003)
39. Reddy, D.R.K., Rao, M.V.S.: Astrophys. Space Sci. **302**, 157 (2006)
40. Lyra, G.: Math. Z. **54**, 52 (1951)
41. Bhattacharya, S., Karade, T.M.: Astrophys. Space Sci. **202**, 69 (1993)
42. Hawking, S.W., Ellis, G.F.R.: The Large Scale Structure of Space–Time. Cambridge University Press, Cambridge (1976)